RHIC/AP/11
September, 1993
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Roll misalignments and skew errors of IR triplet quads

Vertical dispersion and linear coupling

The differential equation for the vertical closed orbit displacement y is

$$y'' + K y = D \tag{1}$$

where a prime indicates differentiation with respect to azimuthal location, D is the inverse of the local bending radius, and K is the local quadrupole strength (positive for vertical focussing). The solution for a point source of strength $\Delta y'$ with periodic boundary conditions is familiar:

$$y = \Delta y' \frac{\sqrt{\beta_y \beta_{y0}}}{2 \sin(\pi Q_y)} \cos(|\Delta\phi| - \pi Q_y) \tag{2}$$

Here $\Delta\phi_y$ is the phase difference between the source and the reference point, and Q_y is the vertical tune. Similarly, the differential equation for the vertical dispersion η_y is

$$\eta_y'' + K \eta_y = (2\theta K + S) \eta_x \tag{3}$$

where θ is the (small) misalignment roll angle of a normal quadrupole, η_x is the horizontal dispersion, and S is the skew quad strength of a correction element. By analogy with equation 2, the less familiar solution to this equation is

$$\eta_y = \theta \frac{\sqrt{\beta_y \beta_{y0}}\ \eta_{x0}}{f \sin(\pi Q_y)} \cos(|\Delta\phi_y| - \pi Q_y) \equiv H_\theta\ \theta \tag{4}$$

for a single thin rolled quad of focal length f, and

$$\eta_y = \frac{1}{F} \frac{\sqrt{\beta_y \beta_{y0}}\ \eta_{x0}}{2 \sin(\pi Q_y)} \cos(|\Delta\phi_y| - \pi Q_y) \equiv H_F \frac{1}{F} \tag{5}$$

for a single thin skew quad of focal length F. The quantities H_θ and H_F defined here are vertical dispersion sensitivity coefficients. If the reference point is at the source point, so that $\Delta\phi_y = 0$ and $\beta_y = \beta_{y0}$, then these coefficients are

$$H_\theta = \frac{\beta_{y0}\,\eta_{x0}}{f} \cot(\pi Q_y) \qquad \text{6a}$$

$$H_F = \frac{\beta_{y0}\,\eta_{x0}}{2} \cot(\pi Q_y) \qquad \text{6b}$$

These coefficients have their ring-wide maxima at the interaction region (IR) quadrupoles.

Roll misalignments of normal quadrupoles and skew quadrupole fields not only lead to vertical dispersion error waves, but also cause linear coupling. A convenient measure of the strength of a coupling source is the minimum fractional tune difference that is possible when it alone is present. It can be shown[1] that the minimum tune split due to a single short quadrupole rotated by a small angle is just

$$\Delta Q_{min} = \left| \theta \, \frac{\sqrt{\beta_{x0}\,\beta_{y0}}}{f\,\pi} \right| \equiv \left| G_\theta \, \theta \right| \qquad 7$$

while the minimum tune split due to a single short skew quad is

$$\Delta Q_{min} = \left| \frac{1}{F} \, \frac{\sqrt{\beta_{x0}\,\beta_{y0}}}{2\pi} \right| \equiv \left| G_F \, \frac{1}{F} \right| \qquad 8$$

The decoupling coefficients G_θ and G_F are also at their largest at the IR quadrupoles.

RHIC numbers in the storage lattice, $\beta^* = 1$ meter

Two of the six IR's in RHIC are tuned to $\beta^* = 1$ meter in the luminosity optics. The quadrupoles in the four IR triplets on either side of these two IR's are liable to cause significant skew quad errors. Each triplet also contains one skew quad corrector connected to an independent power supply. Table 1 show various lattice properties at the center of these elements, including the H and G sensitivity coefficients. Note that H_θ and G_θ definitions apply to the normal quads, and H_F and G_F apply to the corrector. Also note that, in practice, triplet quads are not short - their physical lengths have the same order of magnitude as their focal lengths. The table shows that triplet quads have skew quad sensitivities that are almost two orders of magnitude larger than the regular arc quadrupole sensitivities.

Name	β_x [m]	β_y [m]	η_x [m]	$\mu_x[2\pi]$	$\mu_y[2\pi]$	L[m]	f[m]	H	G
Quad 1	718	668	.520	.2448	.2433	1.44	12.03	44	18.3
Quad 2	1354	550	.731	.2455	.2444	3.40	−5.25	−117	−52.3
Quad 3	575	1313	.490	.2463	.2453	2.10	8.46	116	32.7
corrector	845	999	.586	.2459	.2451			446	146.2
F quad	49.6	9.8	1.843			1.11	−11.02	−2.5	−.6
D quad	10.4	48.6	.940			1.11	10.66	6.5	.7

Table 1. Optical properties of the IR triplet quadrupoles and the local skew quad corrector, in the low beta storage lattice. Parameters for arc F (horizontally focussing) and D (vertically focussing) quadrupoles are included for comparison. Positive f implies vertical focusing. The vertical tune is taken to be $Q_y = 29.185$.

Table 1 also leads to the conclusion that linear coupling is a much more serious problem than vertical dispersion generation. For example, it would be convenient to set tolerances on the misalignment roll angle θ so that

1 the minimum tune split caused by one triplet quadrupole is much less than the nominal tune split of $\Delta Q = 0.01$ (nominal RHIC tunes are $Q_x = 28.190$, and $Q_y = 29.180$), and

2 the vertical dispersion created is much less than 0.1 meters.

Taking quad 2 as the worst case example, the minimum tune split condition leads to a requirement that $\theta << 0.19$ milliradians, while the vertical dispersion condition requires $\theta << 0.85$ milliradians. The first of these two numbers is practically unattainable. A natural conclusion is that it will be necessary to correct local coupling sources in RHIC triplets, after global linear decoupling has been achieved using two families of skew quadrupoles. Fortunately it turns out that vertical dispersion is a convenient diagnostic in adjusting the strength of the skew quadrupole corrector in each low beta triplet.

A numerical experiment demonstrating local triplet correction

Table 2 shows the results of a numerical experiment that was performed to test out the quantitative predictions derived above. The results in the far right column agree well with the predictions of the G column of Table 1, except in the fifth row, where the tune split is so large that one of the eigentunes approaches an integer resonance. The measured vertical dispersions at the locations of each single source are also in good agreement with the H column of Table 1. It is worth noting that the vertical dispersion is not affected by the fact that all these "machines" are fully coupled - except, again, when an integer resonance is approached.

θ_1[mr]	θ_2[mr]	θ_3[mr]	F_c^{-1}[km^{-1}]	η_{y1}[m]	η_{y2}[m]	η_{y3}[m]	$\eta_{y\,corr}$[m]	ΔQ
.0	.0	.0	.0	.0	.0	.0	.0	.0
1.0	.0	.0	.0	.044	.040	.063	.055	.018
.0	1.0	.0	.0	−.138	−.125	−.193	−.168	.052
.0	.0	1.0	.0	.085	.077	.118	.104	.033
.0	.0	.0	−1.0	−.718	−.649	−1.001	−.872	.157
1.0	−1.0	1.0	.0	.339	.308	.474	.414	.106
1.0	−1.0	1.0	−.707	−.004	−.002	−.004	−.003	.0

Table 2. Results of a numerical experiment, adding selected errors to elements in one low beta triplet of the RHIC lattice. Horizontal and vertical fractional tunes were adjusted to be exactly Q = .185, so that $\Delta Q = \Delta Q_{min}$ is due solely to coupling.

The last two rows show that the single skew quad corrector is very effective in compensating for the local coupling caused by 1 milliradian angles in all three triplet quadrupoles. This works because there is essentially no phase advance across the triplet, as Table 1 shows. Better still, the vertical dispersion is (almost) perfectly compensated by the same corrector setting. This shows that the corrector setting may be properly set to remove local coupling by observing vertical dispersion. There is a dual plane beam position monitor on each end of every triplet in RHIC, available to make such measurements. Technically, the correct observable is not the total vertical dispersion, but the variation of vertical dispersion with β^{*-1}, that is, the vertical dispersion coming from the triplet. However, the role of the low beta triplets is liable to be so strong that it is probably merely an apple polishing nicety to subtract the background dispersion contribution due to the rest of the ring.

References

1 S. Peggs, "Coupling and Decoupling in Storage Rings", IEEE Trans. Nucl. Sci., Vol. NS-30, No. 4, p. 2460, August 1983